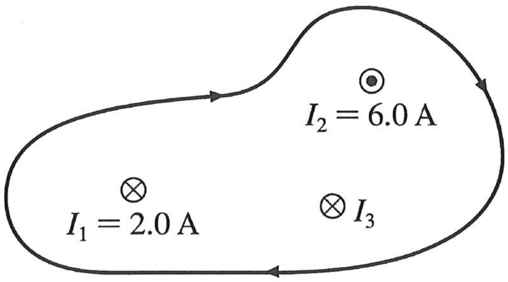
1. The value of the integral $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\mathrm{encl}}$ around the closed path in the figure below is 3.77×10^{-6} T m. What is the value of I_3?



$I_{encl} = I_1 + I_2 + I_3$

$= 2A - 6A + I_3$

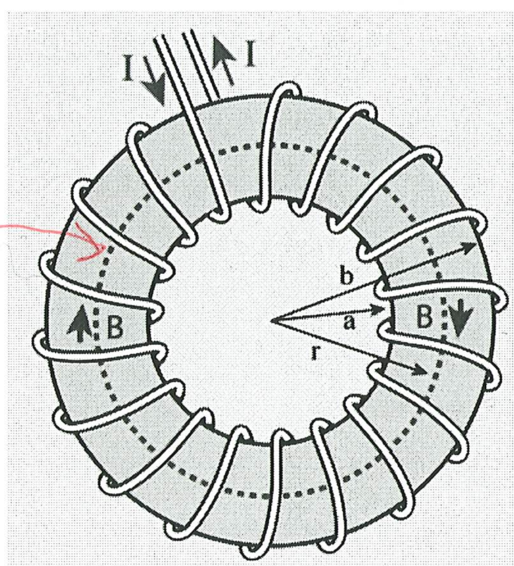
$\therefore \mu_0(-4A + I_3) = 3.77 \times 10^{-6}$ Tm

$\therefore I_3 - 4A = 3A$

$\therefore I_3 = 7A$

2. A toroid consists of a coil of N turns wrapped around a doughnut-shaped former. Use Ampère's law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\mathrm{encl}}$ to find the strength of the magnetic field B a distance r away from from the centre of the toroid (see the figure below). Your answer should be expressed in terms of N, r, and the current I passing through the coil of wire.

Amperian Loop.



By symmetry, $\vec{B} \parallel d\vec{l}$ around amperian loop & B is const. everywhere around loop.

$\therefore \oint \vec{B} \cdot d\vec{l} = \oint B\,dl = B \oint dl = 2\pi r B$

$= \mu_0 I_{encl}$

$I_{encl} = NI$

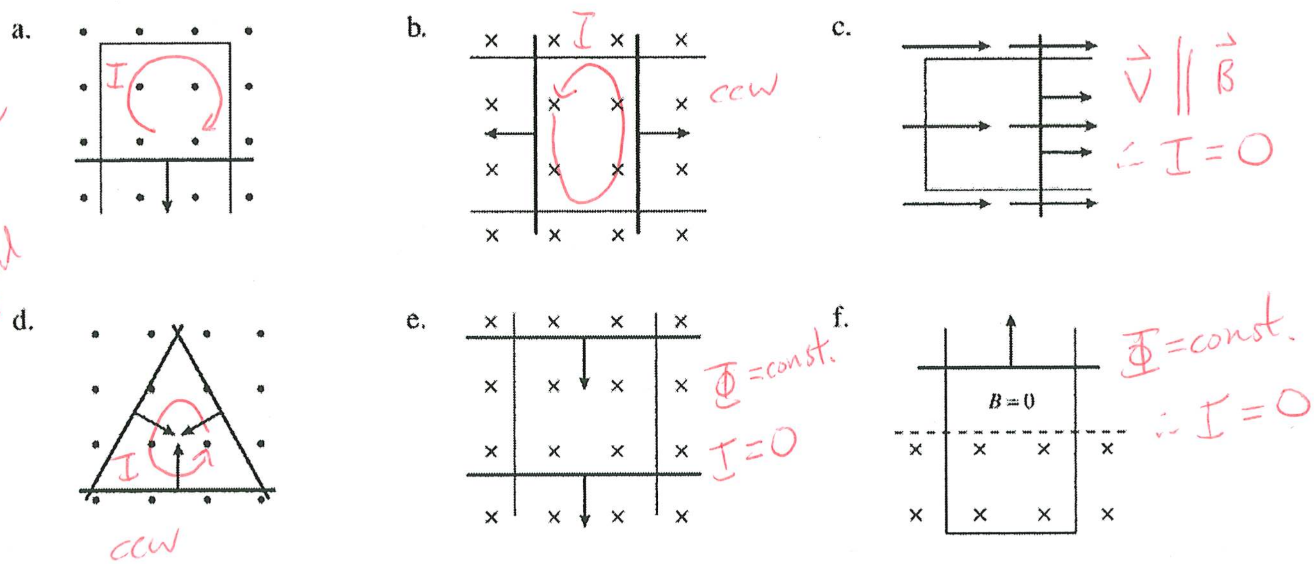
↑ no. of turns around toroid.

$\therefore 2\pi r B = \mu_0 N I$

$B = \dfrac{\mu_0 N I}{2\pi r}$

3.

The figures below show one or more metal wires sliding on fixed metal rails in a magnetic field. For each, determine if the induced current flows clockwise, flows counterclockwise, or is zero. Show your answer by drawing it.

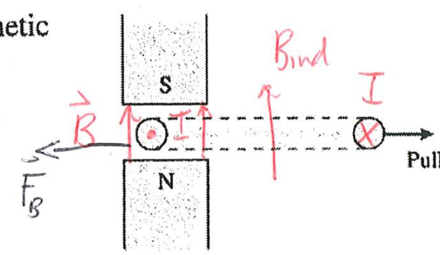


Induced current creates a $\vec{B}_{ind}$ that opposes change in flux.

4.

A loop of copper wire is being pulled from between two magnetic poles.

a. Show on the figure the current induced in the loop. Explain your reasoning.



Φ through loop is decreasing.
$\therefore I_{ind}$ creates a $\vec{B}_{ind}$ that is in same dir'n as $\vec{B}$

b. Does either side of the loop experience a magnetic force? If so, draw and label a vector arrow or arrows on the figure to show any forces.

Left side experiences a force

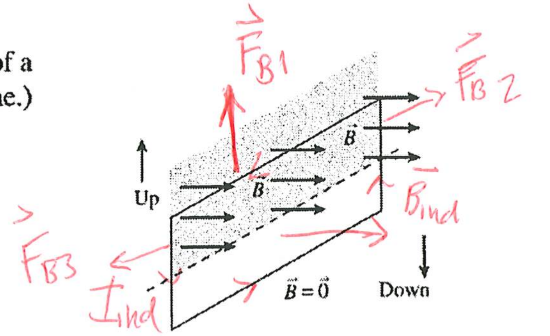
$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$

By RHR dir'n of $\vec{F}_B$ to left

5\. A vertical, rectangular loop of copper wire is half in and half out of a horizontal magnetic field. (The field is zero beneath the dashed line.) The loop is released and starts to fall.

a. Add arrows to the figure to show the direction of the induced current in the loop.

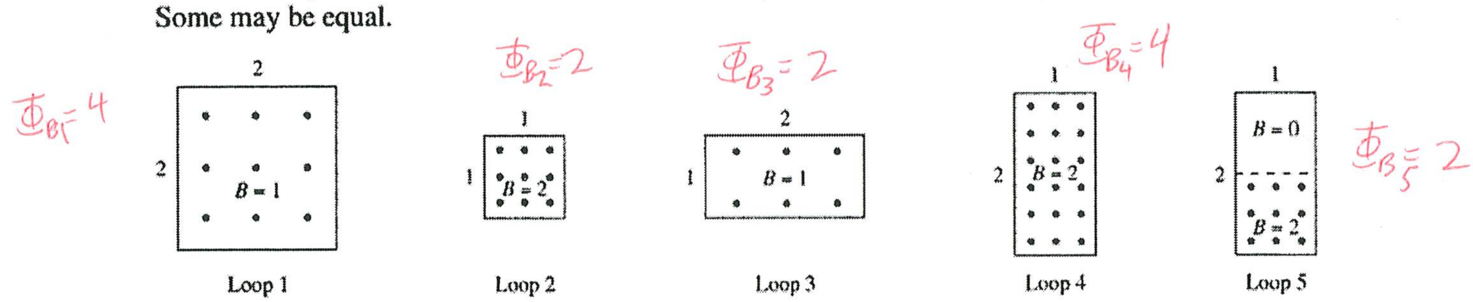
b. Is there a net magnetic force on the loop? If so, in which direction? Explain.



Current in top wire experiences an upward force by RHR.

Side wires experience forces that cancel one another.

6\. The figure shows five loops in a magnetic field. The numbers indicate the lengths of the sides and the strength of the field. Rank in order, from largest to smallest, the magnetic fluxes Φ_1 to Φ_5. Some may be equal.



Order:

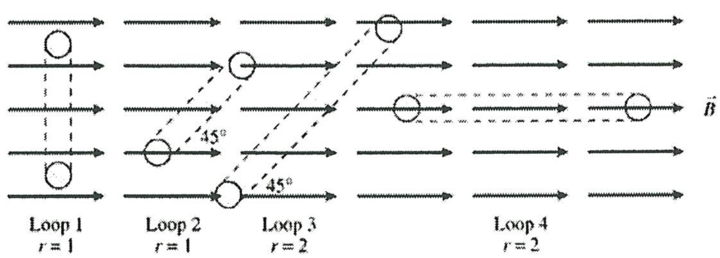
Explanation:

For uniform $\vec{B}$, $\Phi_B = BA$

$\Phi_{B4} = \Phi_{B1} > \Phi_{B2} = \Phi_{B3} = \Phi_{B5}$

7.

The figure shows four circular loops that are perpendicular to the page. The radius of loops 3 and 4 is twice that of loops 1 and 2. The magnetic field is the same for each. Rank in order, from largest to smallest, the magnetic fluxes Φ_1 to Φ_4. Some may be equal.



$\Phi_{B1} = B\pi r^2$

$\Phi_{B2} = B\pi r^2 \cos 45 = \frac{1}{\sqrt{2}} B\pi r^2$

$\Phi_{B3} = B\pi (2r)^2 \cos 45 = \frac{4B\pi r^2}{\sqrt{2}} = 2\sqrt{2}B\pi r^2$

$\Phi_{B4} = 0$

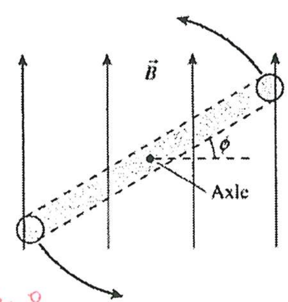
Order:

Explanation:

$\Phi_{B3} > \Phi_{B1} > \Phi_{B2} > \Phi_{B4}$

8.

A circular loop rotates at constant speed about an axle through the center of the loop. The figure shows an edge view and defines the angle ϕ, which increases from 0° to 360° as the loop rotates.



a. At what angle or angles is the magnetic flux a maximum?

ϕ max at 0 & 180°

b. At what angle or angles is the magnetic flux a minimum?

ϕ min at 90° & 270°

c. At what angle or angles is the magnetic flux *changing* most rapidly? Explain your choice.

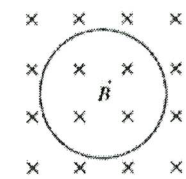
$\Phi = BA\cos\phi = BA\cos\omega t$ ($\omega t = \phi$ where ω is rotational speed)

$\frac{d\Phi}{dt} = -\omega AB \sin\omega t = -\omega BA\sin\phi$

$\sin\phi$ is max when $\phi = 90°$ & $270°$

9.

Does the loop of wire have a clockwise current, a counterclockwise current, or no current under the following circumstances? Explain.



a. The magnetic field points into the page and its strength is increasing.

B_{ind} opposes B.

$\therefore$ I is ccw

b. The magnetic field points into the page and its strength is constant.

since B = const, $\frac{d\Phi_B}{dt} = 0$ & $I_{ind} = 0$

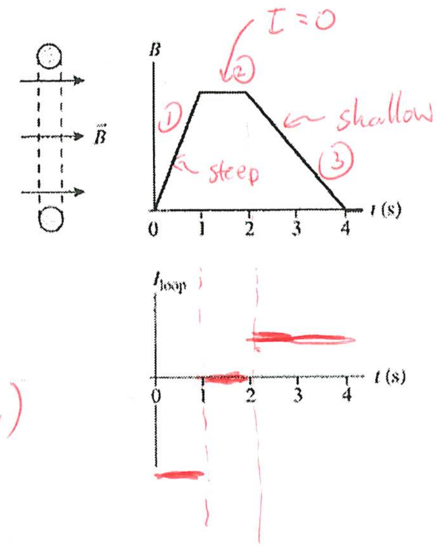
c. The magnetic field points into the page and its strength is decreasing.

B_{ind} in dir'n B

I_{ind} is cw

10.

A loop of wire is perpendicular to a magnetic field. The magnetic field strength as a function of time is given by the top graph. Draw a graph of the current in the loop as a function of time. Let a positive current represent a current that comes out of the top and enters the bottom. There are no numbers for the vertical axis, but your graph should have the correct shape and proportions.



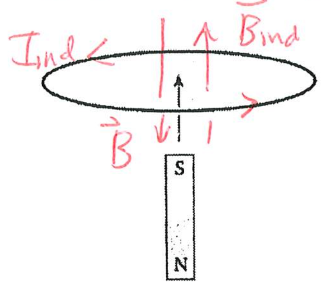
① $\frac{dB}{dt}$ is large. B increasing, ∴ $\vec{B}_{ind}$ opposes $\vec{B}$ (I is negative)

② $\frac{dB}{dt} = 0$ ∴ I = 0

③ $\frac{dB}{dt}$ is small. B decreasing. ∴ $\vec{B}_{ind}$ in same dir'n as $\vec{B}$ (I is positive)

11.

A loop of wire is horizontal. A bar magnet is pushed toward the loop from below, along the axis of the loop.

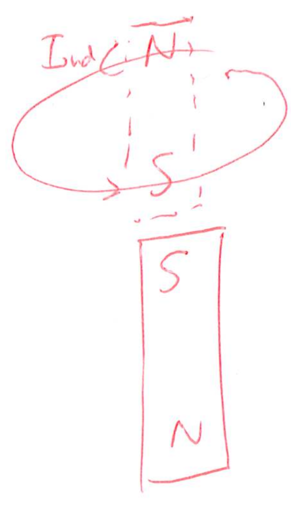


a. What is the current direction in the loop as the magnet is approaching? Explain.

B is increasing. $\vec{B}_{ind}$ opposes $\vec{B}$

I ccw looking from above.

b. Is there a magnetic force on the loop? If so, in which direction? Explain. Hint: A current loop is a magnetic dipole.

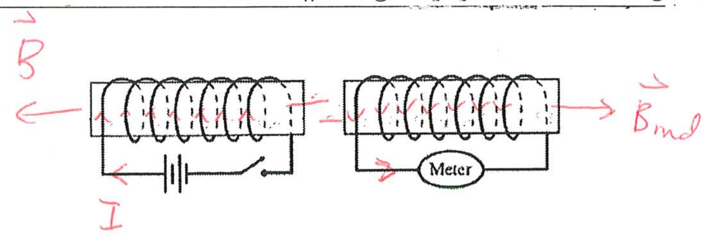


South poles repel.

Force on loop is up.

12.

a. Just after the switch on the left coil is closed, does current flow right to left or left to right through the current meter of the right coil? Or is the current zero? Explain.



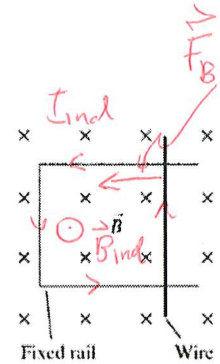
$\vec{B}_{ind}$ in right coil opposes $\vec{B}$ due to left coil. By RHR, I_{ind} is left to right through meter.

b. Long after the switch on the left coil is closed, does current flow right to left or left to right through the current meter of the right coil? Or is the current zero? Explain.

A long time after switch closed, I is const & B is const in left coil. ∴ $\frac{d\Phi}{dt} = 0$ in right coil & $I_{ind} = 0$.

13.

A metal wire is resting on a U-shaped conducting rail. The rail is fixed in position, but the wire is free to move.



$\vec{F}_B = I\vec{\ell} \times \vec{B}$

a. If the magnetic field is increasing in strength, does the wire:

- i. Remain in place?
- ii. Move to the right? (b)
- (a) iii. Move to the left?
- iv. Move up on the page?
- v. Move down on the page?
- vi. Move out of the plane of the page, breaking contact with the rail?
- vii. Rotate clockwise?
- viii. Rotate clockwise?
- ix. Some combination of these? If so, which?

Explain your choice.

$\vec{B}_{ind}$ opposes $\vec{B}$ s.t. I_{ind} is ccw. current in rail experiences a magnetic force due to $\vec{B}$. $\vec{F}_B$ is to left.

b. If the magnetic field is decreasing in strength, which of the above happens? Explain.

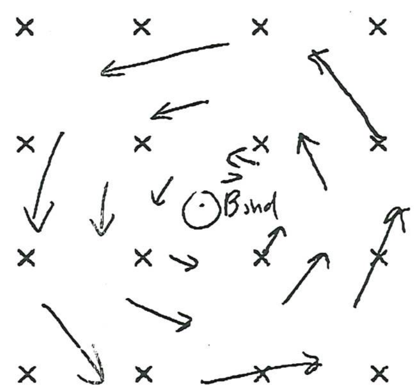
I_{ind} would reverse dir'n & which would reverse dir'n of $\vec{F}_B$

$\vec{F}_B$ is to right.

14.

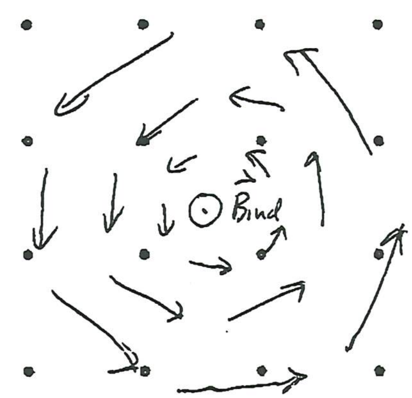
Consider these two situations:

a. Draw the induced electric field.



$\vec{B}$-field rapidly increasing

b. Draw the induced electric field.



$\vec{B}$-field rapidly decreasing